A MATHEMATICAL MODEL FOR PREDICTING THE DAMAGE RATE OF A COCOYAM CHIPPER

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ABSTRACT

A mathematical expression for predicting the damage rate for chipping process of a mechanically operated cocoyam chipper is presented. The model for the damage rate was developed by dimensional analysis, using the concept of Buckingham's Pi Theorem. The model was verified and validated by fitting it into experimental data from the developed mechanical cocoyam chipper. The R^2 value of goodness of fit and its significance level for the compared predicted and measured chip damage data obtained were evaluated. Also, the statistical significance test at 5% level was carried out by computing the student's t- test and then determining the significance level in a table t – values. The results obtained revealed that the fitted model correlated well with the experimental data with R-square value of 0.98. Also, the difference between the means of the predicted and the measured damage rate was not statistically significant at 5% level since the calculated t - value of 0.103 obtained is less than the table t – value of 2.306.

Key words: Model, Predicting, Damage rate, Cocoyam, Chipper

1.0 INTRODUCTION

The operational damage from crop chipping processes refers to the mechanical and material losses that occurs during size reduction for crops such as cocoyam, cassava, yam etc. The major limiting factor in the utilization of cocoyam is the presence of oxalates which causes irritation when foods prepared from them are eaten (Sefa-Dedeh and Agyir-Sackey, 2004). Cocoyam processing helps to reduce the oxalate content and produce good quality flour (Onayemi and Nwigwe, 1987). Chipping operation is one of the most important post-harvest processes for root and tuber crops which help to achieve the required size reduction for easy drying of the crops (Ikejiofor, *et al.* 2016). Manual chipping operation is laborious and time consuming. It is necessary to mechanize the chipping process of cocoyam in order to overcome the difficulties of handling large scale processing with traditional method. The various factors affecting the performance of chipping machine as presented by Adejumo *et al.* (2011), Bolaji *et al*. (2008), Ogundipe *et al*. (2011), and others include machine factors such as: cutting velocity, feed rate, chipping slot size, weight of chipping disc, chipping force, radius of chipping plate, feeding chute clearance and also crop factors such as: crop moisture content, bulk density, length and weight of tubers. These factors if not properly controlled could reduce the performance of the chipping machine. Therefore, modeling the performance parameters or contributing factors for chipping process of cocoyam would provide better understanding of the fundamental relationship of these machine and crop variables in order to identify the contribution of each variable (Ikejiofor, *et al.* 2016). Previous studies on modeling of agricultural products production and processing operations exist, but rare on cocoyam. These include mass and volume modeling of the tubers by Raji and Oyefeso (2010).

The mathematical model for predicting grain damage of stationary grain thresher was developed by Ndirika (2003). The probabilistic theory and dimensional analysis using Buckingham's Pi theorem was used in modelling the grain damage. Modelling of grain breakage in a vertical rice mill was reported by Yong Zeng *et al.* (2022). Also, the mathematical model for forecasting product losses in crop production projects was also reported by Tryhuba, *et al.* (2021). A mathematical model for predicting the mechanical damage of tomato was also reported by Zhiguo *et al*. (2017). There is great need to develop models for predicting the performance of cocoyam chipper in order to establish optimum condition for the construction and operation of the machine. Previous studies on the evaluation of various cassava chipping machines by Ojomo *et al.* (2011), Ogundipe *et al*. (2011), Raji and Igbeka (1994), Bolaji *et al*. (2008) and others reported that the following performance parameters: chipping efficiency, throughput capacity, power requirement and chip damage are required in evaluating the chipping performance. These parameters have to be studied in order to provide better understanding of the fundamental relationships for the different machine and crop variables such as: moisture content, tuber length, feed rate, cutting velocity, feeding chute clearance / angle, radius of chipping plate, weight / thickness of plate and chipping slot size. To develop an efficient cocoyam chipping machine, the variables that can influence the rate of chip damage should be considered. Ikejiofor, (2017) indicated variables of importance that can affect cocoyam chip damage rate to include tuber average length L_a, bulk density β , cutting velocity V_c, feed rate F_r and chipping slot size S_s, The purpose of this study was to develop and verify mathematical model for predicting the rate of chip damage of a cocoyam chipper.

2.0 MATERIALS AND METHODS

2.1 Theoretical Development

The rate of chip damage can be determined by the use of dimensional analysis. Assuming that the variables of importance are tuber average length L_a, bulk density β , cutting velocity V_c, feed rate F_r and chipping slot size S_s, as in equation 1:

$$D_c = f(L_a, \beta, V_c, F_r, S_s) \quad (1)$$

Using the [M], [L], [T] system of dimension (Ikejiofor, 2017), the dimensions of the variables identified in this study and the dimensional matrix are presented in Table 1 and 2 respectively. To identify the dimensionless group to be formed, the Buckingham's Pi theorem was applied as follows:

The dependent variable = D_c

The repeating variables = L_a, β , V_c, F_r, S_s

Total number of variables = 6

Table 1: Dimensions of the variables influencing (D_c)

Variable	Symbol	Unit	Dimension [M], [L], [T]
Chip damage rate	D_c	kg/hr	MT^{-1}
Tuber average length	L_a	m	L
Crop bulk density	β	kg/m^3	ML^{-3}
Cutting velocity	V_c	m/s	LT^{-1}
Feed rate	F_r	kg/s	MT^{-1}
Chipping slot size	S_s	m^2	L^2

Table 2: Dimensional matrix of the variables

Parameters **Dimensions**	**D_c**	**L_a**	β	**V_c**	**F_r**	**S_s**
M	1	0	1	0	1	0
L	0	1	-3	1	0	2
T	-1	0	0	-1	-1	0

Number of variables = 6
Number of fundamental dimensions = 3
Number of dimensionless groups to be formed = 6 – 3 = 3
The required solution will be as eqn 2:

$$\pi_1 = K_d\, f(\pi_2, \pi_3) \qquad (2)$$

Where,
π_1, π_2 and π_3 = First, second and third pi terms respectively
K_d = damage constant
Using the Buckingham's Pi theorem, π_1, π_2 and π_3 were found to be:

$$\pi_1 = \frac{D_c}{F_r} \qquad (3)$$

$$\pi_2 = \frac{L_a}{\sqrt{S_2}} \qquad (4)$$

$$\pi_3 = \frac{\beta V_C S_s}{F_r} \qquad (5)$$

Let $\pi 4 = \pi 2 \times \pi 3$

$$\pi 4 = \frac{L_a}{\sqrt{S_s}} \times \frac{\beta V_c S_s}{F_r} = \frac{L_a \beta V_c S_s}{\sqrt{S_s} F_r}$$

$$\pi 4 = \frac{L_a \beta V_c S_s}{\sqrt{S_s} F_r} = \left[\frac{(L_a \beta V_c)^2 S_s}{F_r{}^2}\right] \qquad (6)$$

$$\therefore \quad \frac{D_c}{F_r} = K_d \left[\frac{(L_a \beta V_c)^2 S_s}{F_r{}^2}\right] \qquad (7)$$

Rearranging equation 7, then

$$D_c = K_d \left[\frac{(L_a \beta V_c)^2 S_s}{F_r} \right] \quad (8)$$

Substituting value of β as $\frac{\beta_d}{1-M_c}$ into equation 5, then

$$D_c == K_d \left[\frac{(L_a \beta_d V_c)^2 S_s}{(1-M_c)^2 F_r} \right] \quad (9)$$

This equation 9 expresses the rate of chip damage for the cocoyam chipper.

2.2 Value of Constant

The damage constant, K_d was determined by linearizing $\pi_1 = \frac{D_c}{F_r}$ and $\pi_4 = \left[\frac{(L_a \beta V_c)^2 S_s}{F_r{}^2} \right]$ from the developed model by method of least square using data from available literatures (Aji *et al*., 2013, Sanmi *et al*., 2004, Oladeji, 2004, Bolaji *et al*., 2008 and Ogundipe *et al*., 2011) as indicated in Table 3. No previous data on cocoyam chipper was available and the existing data available were mostly for cassava chippers. Ikejiofor and Ndirika (2015) reported on the determination of the physical properties of two cocoyam varieties, and the average length of tubers and the bulk density wet basis determined were 0.1004m and 485.77kg/m^3 respectively. These values were used in the determination of π_4 using equation 6. Also, the π_1 values were determined using equation 3. The data generated for π_1 were plotted against the π_4 in Figure 1 and the slope of the line gives the value of the damage constant, K_d which was found to be 9.328x10^{-5}.

Table 3: Data used to determine K_d Values

D_c (kg/s)	La (m)	β (Kg/m^3)	V_c (m/s)	S_s (m^2)	F_r (Kg/s)	π_1	π_4	Chipper Type and Source
0.0041	0.1004	485.77	2.420	0.00030	0.0924	0.0444	489.48	Cassava Chipper (Aji *et al*., 2013)
0.0067	0.1004	485.77	5.97	0.000057	0.0667	0.1004	1086.18	Cassava Chipper (Sanmi *et al*., 2013)
0.0142	0.1004	485.77	4.713	0.00032	0.1104	0.1286	1387.19	Cassava Chipper (Oladeji. 2004)
0.0104	0.1004	485.77	6.284	0.000092	0.0771	0.1349	1453.72	Cassava Chipper (Bolaji *et al*., 2013
0.0186	0.1004	485.77	7.86	0.00013	0.0926	0.2009	2227.89	Cassava Chipper (Ogundipe *et al*., 2011)

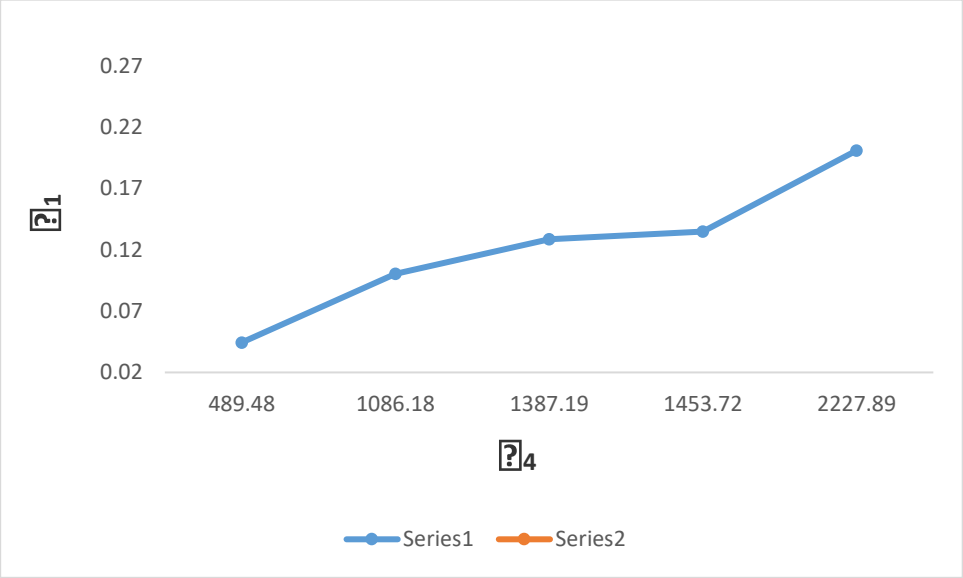


Figure 1: Determination of the damage constant, K_d

2.3 Verification and Validation of the Model

The developed chip damage rate model was verified in order to confirm its consistency with established experimental results from a cocoyam chipper. The study was conducted using the experimental data from the developed cocoyam chipper which was then compared with the data from the predicted model. The crop and operating conditions for the cocoyam chipper used are presented in Table 4. A model would have greater confidence if a good fit and a high significance level is attained. However, the method developed by Gregory and Fedler (1986) for calculating the coefficient of determination, R^2 statistically for non- linear as well as for linear function and with one or more independent variables was adopted. Thus,

$$R^2 = \frac{1 - V_o}{V_t} \qquad (10)$$

Where,

R^2 = coefficient of determination

V_o = estimated variance about the mean from the measured data

V_t = estimated variance about the mean from the predicted model

The statistical significance test for the R^2 value obtained was carried out by computing the "t" test using the method developed also by Obi (2002), and then determining the significance level in a table of t – values.

$$T_{cal} = \frac{\overline{X}_A - \overline{X}_B}{S_{\bar{d}}} \qquad (11)$$

Where, $\bar{X}_A$ = mean of first sample

$\bar{X}_B$ = mean of second sample

S_d = The Standard Error of the difference between two means

$$S_d = \sqrt{\frac{2S^2}{r}} \qquad (12)$$

Where, r = number of replications

$$S^2 = \frac{\sum_{i=1}^{n} X_i^{\,2} - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}}{n-1} \qquad (13)$$

Where, n = sample number

Table 4: Crop and operating conditions for the chipper

		Values / Levels		
S/N	**Parameters**	**1**	**2**	**3**
1	Cutting Velocity, V_c (m/s)	3.93	4.58	4.91
2	Av. Length of Tubers, La(m)	0.0872	0.1006	0.1134
3	Feed Rate, Fr (kg/s)	0.056	0.058	0.060
4	Chipping Slot Size, Ss (m^2)	0.0014	0.0016	0.0018
5	Moisture Content, M_c (%)	57.81	52.39	46.97
6	Bulk density, β wet basis (kg/m^3)	474.12	485.77	497.41

2.4 Experimental Samples

The cocoyam varieties, NXs001 and NXs002 with the average length, bulk density and moisture content of 0.1004m, 485.77 kg/m^3 and 52.39% respectively were used for the experiment. The tubers were obtained from National Root Crops Research Institute farm. Before the chipping process, the harvested tubers were peeled and washed thoroughly.

2.5 Instrumentation and Measurements

The developed cocoyam chipper used is a disc type with circular stainless-steel chipping plate having sharp edged slots for the chipping. The chipper is powered by 2 hp electric motor. The data generated from the developed cocoyam chipper used in the verification of the chip damage rate model were evaluated by the following methods:

Chip damage rate measurement: The chip damage rate was measured by weighing the quantity of broken and crushed chips recovered from the machine per unit time, expressed in kg/s.

Weight measurement: An electronic balance of sensitivity 0.01g and the range of 0.01g to 5000g was used for weighing.

Time measurement: Time was measured using a stop watch.

Moisture contents determination: Moisture content of the sliced cocoyam corms were determined by oven drying at a temperature of 75°C for 24 hours. The percentages of moisture content wet basis were calculated as according to Kajuna *et al*., (2001).

Bulk density determination: The bulk density was determined as the ratio between the mass of the cocoyam corms in a container to its volume (Kaleemullah and Kailappan, 2003).

3.0 RESULTS AND DISCUSSIONS

3.1 Model Validation

The chip damage rate model (Equation 8) was validated with measured data from the developed cocoyam chipper. The experiment was conducted using five cutting velocities (3.60. 3.93, 4.25, 4.58 and 4.91m/s) and each run were replicated three times for the three different values of tuber length, La (0.0872, 0.1006 and 0.1134m), bulk density, β (474.12, 485.77 and 497.41 kg/m^3), chipping slot size, S_s (0.0014, 0.0016 and 0.0018m^2), and feed rate, F_r (0.056, 0.058 and 0.060 kg/s). The average data generated which was used for comparison between the values of chip damage rate obtained from the experimental result and that obtained using the formulated performance models are presented in Table 5 and Figure 2. From the result, it was established that the model described well with the measured data obtained from the developed chipper with R^2 value of 0.98. When the means of the predicted and measured chip damage rate were compared statistically, it was revealed that there was no significant difference between the means at 5% level of significance, since the calculated t - value (0.103) is less than the table t - value (2.306). The result reveals a high correlation of the model with data from the developed cocoyam chipper.

Table 5: Computed and measured chip damage rate for the cocoyam chipper

Cutting Velocity (m/s)	La (m)	β (Kg/m^3)	S_s (m^2)	F_r (kg/s)	Computed Chip Damage Rate (kg/s)	Measured Chip Damage Rate (kg/s)
3.60	0.0872	474	0.0018	0.052	0.00493	0.00502
3.93	0.1006	486	0.0016	0.056	0.00618	0.00624
4.25	0.1134	497	0.0014	0.057	0.00743	0.00765
4.58	0.1006	486	0.0016	0.058	0.00839	0.00856
4.91	0.1134	497	0.0018	0.060	0.00964	0.00970

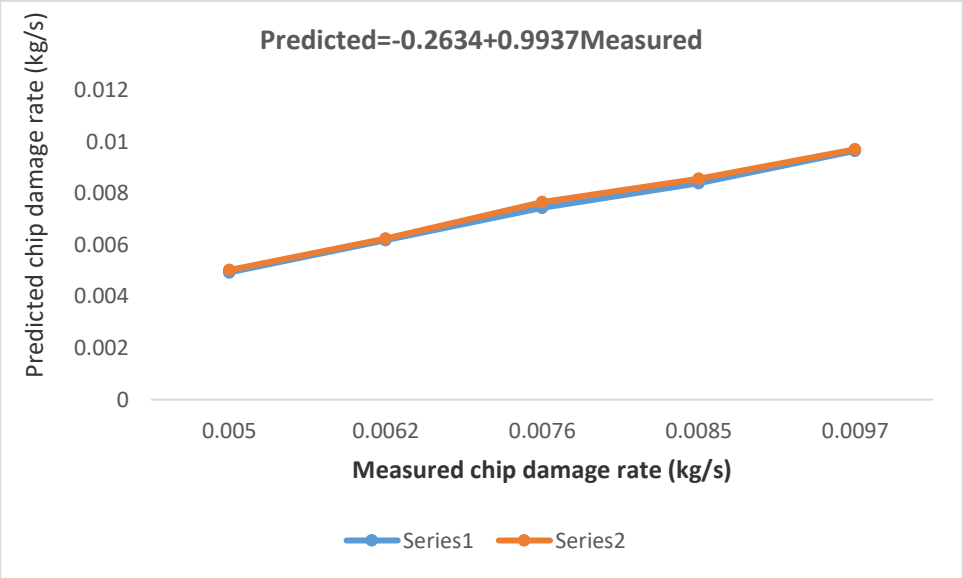


Figure 2: Predicted versus measured chip damage rate

4.0 CONCLUSIONS

The chip damage rate of cocoyam chipping machine can be described using the developed mathematical model. The important variables used for the prediction includes cutting velocity, feed rate, chipping slot size, crop bulk density, tuber length. The data used for the crop bulk density wet basis (474.12. 485.77and 497.41kg/m^3) and tuber length (0.0872, 0.1006 and 0.1134m) were obtained from the cocoyam variety NXs 001 and NXs 002. The difference between the means of the predicted and measured chip damage rate was not statistically significant at 5% level since the calculated t - value (0.103) is less than the table t - value (2.306). Therefore, the developed predicted chip damage rate model compared favourably with the experimental data from the developed cocoyam chipper and cocoyam varieties within the stipulated range are recommended.

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