MATHEMATICAL IMPLEMENTATION OF EFFECT OF INCREASE IN DIVERGENCE ANGLE OF A DIFFUSER ON SPEED AND POWER GENERATION OF HAWT USING DISK THEORY

[1]Otor, Emmanuel and [2]Uju, I. U
Emnico Resources and Engineering Services, Nnewi, Anambra State.
Department of Electrical/Electronic Engineering,
Chukwuemeka Odumegwu Ojukwu University, Uli, Anambra State, Nigeria.
Email: emma4goodness@gmail.com, ujuisidore@yahoo.co.uk

Abstract

*Harnessing wind energy to turn a turbine for power generation requires concentration of the wind at high velocity to give maximum and steady power output. Mathematical relationships are used to analyze the variation of the pressure and velocity of the flow regime in the conical duct. The pressure and velocity magnitude varies oppositely, pressure increases as velocity decreases. This revealed that power extracted by horizontal axis wind turbine (**HAWT**) is the product of the volume flow rate of air through the turbine and the pressure drop across the blades, which is a result of the thrust of the blades. The mathematical model uses disc theory to predict appropriate position of rotor for maximum power production. The diffuser angles considered are 4^0 8^0 12^0 16^0 and 20^0. Energy conversion starts at a certain angle say 4^0 and will increasing until a certain angle of about 20^0 and it will begin decline. As the angles are increased the power output of the turbine also increased until when it get to angle 20^0 then it begin to decline. The turbine operation exhibits a reasonable range of tip speed ratio and high efficiency. The proposed method suggests that an increase in CP of about 0.035, which is about a 27%, for open angle of 12^0, 32% for open angle of 16^0 and 38% for open angle 20^0 increase in aerodynamic efficiencies, were attainable with the DAWT concept. This improvement is primarily due to efficient extraction of energy (wind) blowing near the hub of the main rotor, but in part also due to the addition of another energy extracting device called diffuser augmented system.*

Keywords: Mathematical Analysis, Disk Theory, Pressure and Velocity Variability, Turbine Power Generation

1.0 Introduction

Wind is one of the most promising among other different types of available renewable energy sources (Vennell, 2013). Within the past decade, considerable research has been carried out to improve on wind turbine design and control for increased power conversion, efficiency and availability. Wind turbine consists of a rotor mounted to nacelle and a tower with two or more blades mechanically connected to an electric generator. The gear box in the mechanical assembly transforms slower rotational speeds (90:1), (16.7 rpm input to 1,500 rpm output for the generator) of the wind turbine to higher rotational speed on the electric generator. The rotation of the electric generator's shaft generates electricity whose output is maintained by a control system. There are two types of design models for wind turbines. The classification is made on the basis of their axis in which the turbine rotates. Horizontal axis wind turbine (HAWT) and vertical axis wind turbine (VAWT). The vertical axis wind turbine (VAWT) is also called Darreus rotor named after its inventor. Vertical axis wind turbine is also a type of wind turbine which the main components are placed at the base of the turbine .The arrangement allows the generator and the gearbox to be located near the ground, thereby facilitating maintenance. Horizontal axis wind turbine has blades that look like a propeller that spin on the horizontal axis. The main rotor shaft and electrical generator are placed at the top of a tower and they point to the wind. Wind turbines operate in two modes namely constant or variable speed. For a constant speed turbine, the rotor turns at a constant angular speed regardless of wind variations. One advantage of this mode is that it eliminates expensive power electronics such as inverters and converters. Its disadvantage however, is that it constrains rotor speed so that the turbine cannot operate at its peak in all wind speeds. For this reason a constant wind speed turbine produces less energy at low wind speeds than does a variable wind speed turbine which is designed to operate at a rotor speed proportional to the wind speed below its rated wind speed. The output power or torque of a wind turbine is determined by several factors. Among them are turbine speed, rotor blade tilt, rotor blade pitch angle, size and shape of turbine, area of turbine, rotor geometry (whether it is a HAWT or a VAWT), and wind speed. Modeling and simulation of the ideal wind turbine aerodynamics was done using MATLAB environment. The analysis covers mainly modeling of air flow in a horizontal axis wind turbine system.

2.0 Methodology

Modeling of flow regime in a Diffuser augmented wind turbine (DAWT) gives a clear understanding of how to maximize power captured in wind turbines.

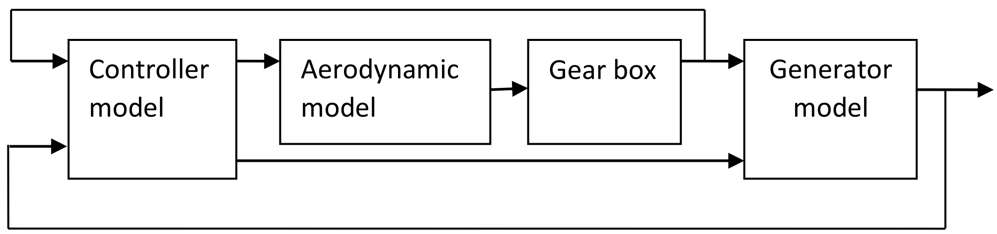


Figure 1: Block diagram of the proposed wind turbine system

Aerodynamic (Blades); is that portion of the system where aerodynamics effect takes place. The components are the blades, either two or more which are attached to a hub, the shaft transfers the mechanical motion of the aerodynamics to the end of the shaft attached to the gear box. The connection between the beginning of the shaft and the end at the gear box is done with the aid of bearings. The effectiveness of the blades depend on a number of factors (1) the tilt angle, this is the angle at which the wind hits the blades, the wider the angle the better the performance of the blades. (2) The swept area; this is the distance from the hub to the tip of the blades. The greater the swept area (the length of the blade) the better the performance of the blades. The Rotor is the length of two opposite blades. The algorithm for conducting the calculation is hereby presented.

2.1 Method for Calculating the Diffuser Effect of Wind Increase at the Entrance to Turbine.

1. Set H and θ_{opt};
2. Calculate r_{opt} according to the formula $r_{optim} = \frac{H}{\pi} x \frac{\pi - \theta_{optim}}{sin\theta_{optim}} = \frac{H}{\pi} x \frac{\pi - 1.1}{\sin - 1.1}$ (1)

3. Calculate angle θ, tilt of the turbine relative to the vertical according to the formula $y = \frac{\frac{Q}{z} - \frac{Q}{2\pi}\theta}{Ws}$ (2)

4. Calculate the radius vector and the polar angle of the point C, the entrance to the turbine according to the formula $ws \times y + \frac{Q}{2\pi}\theta = const$ (3)

5. Calculate diffuser coefficient konf (The degree of convergence or the coefficient of wind amplification in diffuser). According to the formula:

$$P_{turb} = \frac{d_m}{d_t}(wkonf - V_3)V_2 = \rho S {V_2}^2 (wkonf - V_3) \quad (4)$$

(The diameter of the power disk)

6. Calculate the height of the turbine installation: hust= $r_{opt}\sin\theta_{opt}$;
7. Calculate the wind speed at the inlet to the turbine: wkonf=konf*ws;
8. From the received wind speed at the input wkonf, calculate the turbine power.
9. Repeating the calculations from step 1, determine the polar angle θ_{opt} of the turbine installation point corresponding to the maximum power.

As seen in the schematic diagram below, the program starts with assuming the fluid velocity of 1 m/s, the swept area of 1 m^2 and the specific mass of fluid of 1 kg/m^3 and making the initial value of loading coefficient of 0. Then it calculates the velocity at rotor, the thrust and the coefficient of performance, until the amount of the loading coefficient of 4, before the program plots the parameters in graphs.

3.0 System Modeling

Model of the *Disk Theory*, obtained by Towler, 2014 is applied. The modeling of the system is presented in Figure 2. Referring to this figure, the location of free stream is represented by notation (0), the location immediately before the rotor is represented by notation (1), the location immediately after the rotor is represented by notation (2) and the location at downstream is represented by notation (3).

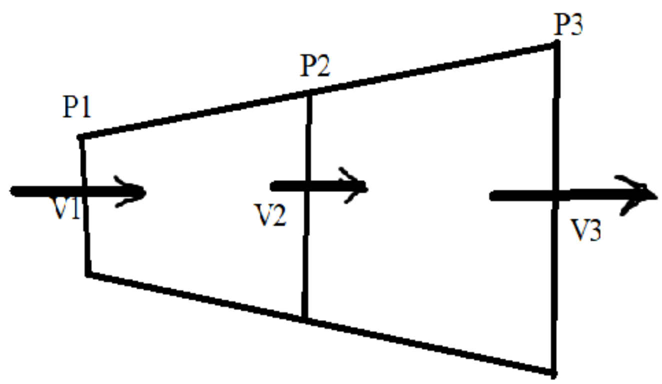


Fig.2: Schematic diagram of air passing through a turbine

klens– Coefficient of pressure reduction at the outlet of the turbine;

1. Section of the air stream at a certain distance from the entrance to the turbine;
2. Working part of the turbine (power disk);
3. Section of the jet at the outlet of the turbine;

P_1− Pressure at the entrance to the power disk;
P_2− Pressure at the output of the power disk;
w_s =Wind speed;
wkonf=w_s×konf − Wind speed, taking into account the diffuser effect at the entrance;
konf – The degree of convergence or the coefficient of wind amplification in diffuser.

The first (internal) task is the creation of a model for the interaction of working blades with air. This model calculates the aerodynamic forces and moments acting on the blades during rotation in the wind field. This model contains a set of kinematic parameters that characterize the motion of the blade relative to air and air relative to the blade. Since both these movements are unsteady (that is, the parameters of these movements changes over time),then in addition to static forces and moments depending on the angle of attack, no stationary forces and moments will act on the blades.

$$P_{turb} = \frac{d_m}{d_t}(wkonf - V_3)V_2 = \rho S{V_2}^2(wkonf - V_3) \qquad (5)$$

V_2− Flow velocity in the working part of the turbine (near the power disk 2);
V_3 − Flow velocity at the outlet of the turbine (cross section 3);
S− Cross − sectional area of the working part (section 2);
ρ− Density of air;
The Bernoulli equation 1-2:(eqn 6), and Bernoulli 2-3(eqn 7), the notation P represent static pressure and the notation V represent the velocity. In this model, it is assumed that the flow of air through the disk experiences a constant velocity (V1=V2) but the static pressure is dropping (P1>P2).Other assumption is that the static pressure of free stream (Po) is assumed to be equaled to that of the downstream (P3).This can be explained that the region within the turbine operating area actually has almost zeroed potential head. Bernoulli's equation theorem can be applied to the tube on either side of the disk.

$$P_\infty + \frac{\rho \times wkonf^2}{2} = P_1 + \frac{\rho {V_2}^2}{2} \qquad (6)$$

The Bernoulli equation 2-3:

$$P_2 + \frac{\rho {V_2}^2}{2} = P_\infty - klens\frac{\rho \times wkonf^2}{2} + \frac{\rho {V_3}^2}{2} \qquad (7)$$

We subtract (7) from (6):

$$\frac{\rho}{2}(wkonf^2 - {V_3}^2)ss = P_1 - P_{2;} \qquad (8)$$

The power of the turbine for the second task is expressed through the pressure drop on the power disk:

$$P_{turb} = (P_1 - P_2)SV_{2;}$$

Considering (6.7):

$$P_{turb} = \left[\frac{\rho}{2}(wkonf^2 - {V_3}^2) + klens\frac{\rho \times wkonf^2}{2}\right]SV_{2;}$$

Considering (5):

$$\rho S{V_2}^2(wkonf - V_3)\left[\frac{\rho}{2}\left(wkonf^2 - {V_3}^2\right) + klens\frac{\rho \times wkonf^2}{2}\right]SV_{2;}$$

Or:

$$(klens + 1) \times wkonf^2 - {V_3}^2 = 2V_2(wkonf - V_3)$$

Or:

$${V_3}^2 - 2V_2V_3 + 2 \times wkonf\left(V_2 - \frac{klens+1}{2}wkonf\right) = 0 \qquad (9)$$

Equation (9) determines the speed V_3 at the turbine outlet for a turbine having the diffuser wind amplifier at the inlet and the diffuser with the interceptor flow accelerator in the working part:

$$V_3 = V_2 \pm \sqrt{{V_2}^2 - 2wkonf\left(\frac{klens+1}{2}wkonf\right)} \qquad (10)$$

Or:

$$V_3 = V_2 \pm \sqrt{(V_2 - wkonf)^2 + klens \times wkonf^2} \qquad (11)$$

If the speed in the working part of the turbine exceeds the speed at the inlet, then the turbine model goes into propeller mode. This state corresponds to the plus sign in expression (11). Since the propeller mode does not interest us, we will take the minus sign.

Expression (10) determines the relationship of the velocities at the inlet, outlet and in the working part of the turbine. For the particular case when the turbine does not have a wind lens at the output (klens = 0), the expression (11) will have the form:

$$V_2 = \frac{wkonf + V_3}{2}$$

If the turbine does not have a wind lens and a diffuser at the input, then expression (11) will have the form :(Klens is the coefficient of pressure reduction at the outlet of the turbine). (Wind lens is the diffuser airflow accelerator through the turbine; it creates a pressure drop at the outlet of the turbine)

$$V_2 = \frac{WS + V_3}{2} \qquad (12)$$

We substitute (3.6) into (3.1) and taking into account that S=4R*h, we obtain:

$$P_{turb} = 4Rh{V_2}^2\left[wkonf - V_2 + \sqrt{{V_2}^2 - 2wkonf\left(V_2 - \frac{klens+1}{2}wkonf\right);}\right] \quad 13$$

According to (3.7), the power coefficient:

$$C_p = \frac{P_{turb}}{P_{ws}} = \frac{2{V_2}^2}{ws^3}\left[wkonf - V_2 + \sqrt{(V_2 - wkonf)^2 + klens}\right] \qquad (14)$$

To determine the speed V_{2opt} at which the power factor reaches its maximum value and the maximum power coefficient, we find the derivative of expression (14) and equate this derivative with zero:

$$\frac{\delta C_p}{\delta V_2} = \frac{2}{ws^3}\left\{\begin{matrix} 2V_2\left[wkonf - V_2 + \sqrt{(V_2 - wkonf)^2 + klens \times wkonf^2}\right] \\ +{V_2}^2\left(-1 + \frac{V_2 - wkonf}{\sqrt{(V_2 - wkonf)^2 + klens \times wkonf^2}}\right) \end{matrix}\right\};$$

Or

$$V_2(2wkonf - 3V_2)\sqrt{(V_2 - wkonf)^2 + klens \times wkonf^2} + 2V_2(V_2 - wkonf)^2 + 2V_2 klens \times wkonf^2$$

Or

$(2wkonf - 3V_2)\left(\sqrt{(V_2 - wkonf)^2 + klens \times wkonf^2 + wkonf - V_2}\right) + 2klens \times wkonf^2 = 0$
15

We transform (13) for the case wkonf = w_s and klens = 0 (turbine without diffuser and without wind lens):
As V_2<wkonf (Turbine operation), then it follows from (15):
$(2w_S - 3V_2)2(w_S - V_2) = 0$, from which follows:

$V_{2_{opt}} = \frac{2}{3}w_s$ 16

For this case, equation (14) has the form:

$C_p = \frac{2V_2{}^2}{ws^3}2(ws - V_2)$ 17

Substituting (16) into (17), we obtain the maximum possible power coefficient:

$C_{pmax} = \left(2\frac{4ws^2}{9} \div ws^3\right)2\left(ws - \frac{2}{3}ws\right) = 0.592$ Betz limit

5 Construction of the configuration of the terrain at the entrance to the turbine for the implementation of the diffuser effect to increase the wind speed.

Consider the air flow diagram shown in Figure 3. The streamline AB represents some relief of the terrain that flows around by the wind. It is clear that the wind speed when flowing over the surface of the relief AB will be greater than on the horizontal terrain before the point A. Thus, if the turbine is installed at any point on the AB line, then the effect of increasing wind speed (diffuser effect) will be realized at the entrance to the turbine. The difficulty is to create a model of such a flow, for calculating the value of the wind speed at each point at the entrance to the turbine. For a single-jet model, we will determine the wind speed at the center of the entrance to the turbine.

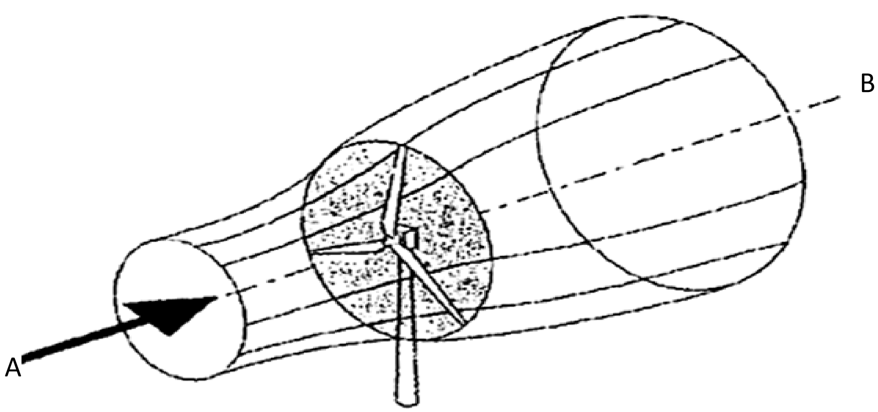
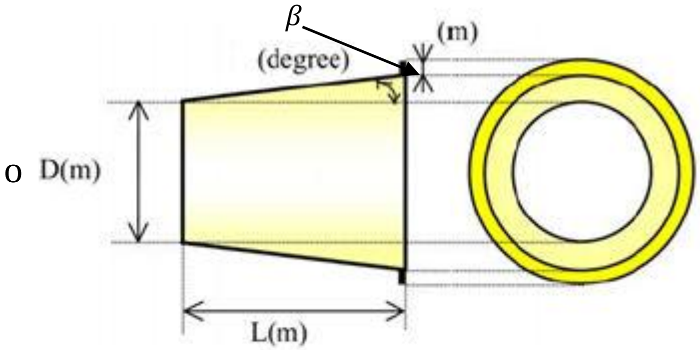


Fig. 3.: Flow simulation scheme of the terrain AB to increase wind speed (diffuser effect).



D(m) entrance diameter

B(degree) expansion or diffuser open angel(Between the first angle D(m) and the second angle B(degree)

L(m) the diffusers main body length (m)splitter open and flange angle.

Figure 4: Interior Shape of a Diffuser

It is clear from the figure 4 that such a flow can be modeled by superimposing the velocity field of horizontal flow on the source velocity field at the origin of the coordinate axis (point O).
The horizontal flow has potential φ and stream function ψ:

$$\varphi = ws \times x;\ \psi = ws \times y$$

The source at the origin has the potential φ and stream function ψ:

$$\varphi = \frac{Q}{2\pi}\ln\sqrt{x^2 + y^2};\ \psi = \frac{Q}{2\pi}\tan^{-}\frac{y}{x}$$

Where,
Q= source flow rate,
Ψ Streamline function
Φ The potential flow
× y The axis of flow
The total flow of the horizontal flow and the source has a potential φ and stream function ψ:

$$\varphi = ws \times x + \frac{Q}{2\pi}\ln\sqrt{x^2 + y^2};\ \ \psi = ws \times y + \frac{Q}{2\pi}\tan^{-}\frac{y}{x}; \qquad (18)$$

The equation of the family of streamlines of the total flow (16) can be written as follows:

$$ws \times y + \frac{Q}{2\pi}\tan^{-}\frac{y}{x} = \text{const} \qquad (19)$$

Or through the polar angle θ:

$$ws \times y + \frac{Q}{2\pi}\theta = const \qquad 20$$

Consider the line of stream that passes through the point A. For this streamline, for y=0 should be θ = π. Substituting this value in (17), find the value of the constant for this streamline:

$$const = \frac{Q}{2}$$

Thus, the equation of the streamline passing through point A will be:

$$ws \times y + \frac{Q}{2\pi}\theta = \frac{Q}{2} \qquad 21$$

tilt angle = $w_s\ x\ y + \frac{Q}{2\pi}\ \theta = \frac{Q}{2}$

$$w_s\, x\, y + \frac{Q}{2\pi} - \left(\frac{Q}{2\pi}\, \theta\right)$$

$y = \frac{\frac{Q}{z} - \frac{Q}{2\pi}\theta}{Ws}$ 22

This streamline consists of two branches: θ = π, which represents the negative semi axis, x, and the curve line AB, the equation of which is obtained if we substitute y = r*sinθ in 3.15 and define or:

$$\text{ws} \times (\text{y} = \text{r} \times \sin\theta) + \frac{\text{Q}}{2\pi}\theta = \frac{Q}{2}$$

or

$r = \frac{Q}{2\pi} \times ws \times \frac{\pi - \theta}{sin\theta}$ 23

Streamline AB can be considered a solid surface, since in a potential flow all the particles of the liquid follow the configuration of the streamlined body. Thus, this flow can be treated as the flow around the terrain AB by the wind at speed w_s. The height of the terrain at any point with the coordinates (r, θ):

$\text{r} \times \sin\theta = \frac{\text{Q}}{2\pi \times \text{ws}} \times (\pi - \theta)$ 24

The maximum height of the elevation of the terrain is obtained from (24), if substitute $\theta = 0$:

$H = \frac{Q}{2ws},\ Q = 2H \times ws$ 25

Substituting the value of the flow rate 2 of source from (3.18) into the expression for the potential (3.15), we find the projections of velocity at any point:

$\varphi = ws\left(x + \frac{H}{\pi} ln\sqrt{x^2 + y^2}\right);$

$V_x = \frac{\delta\varphi}{\delta x} = ws\left(1 + \frac{Hx}{\pi x^2 + y^2}\right) = ws\left(1 + \frac{H}{\pi}\frac{cos\theta}{r}\right)$ 26

$V_{y=} \frac{\delta\varphi}{\delta x} = ws\left(1 + \frac{Hy}{\pi x^2 + y^2}\right) = ws\left(1 + \frac{H}{\pi}\frac{sin\theta}{r}\right)$ 27

If we substitute in these expressions the magnitude of the radius vector r from the streamline AB equation, then we obtain the velocity projections at any point of the line AB:

$V_x{}^{AB} = ws\left(1 + \frac{sin\theta cos\theta}{\pi - \theta}\right)$ 28

$V_x{}^{AB} = ws\ \frac{sin^{2\theta}}{\pi - \theta}$ 29

The flow velocity at any point of the AB line:

$V_x{}^{AB} = ws\sqrt{1 + \frac{sin^{2\theta}}{\pi - \theta}\frac{sin^{2\theta}}{(\pi - \theta)^2}}$ 30

To determine the optimum angle θ, at which the speed *ø: reaches a maximum, we find the derivative with respect to the angle θ of Expression 26 and equate it to zero:

$$\cos 2\theta\ (\pi - \theta)^2 + sin2\theta\ (\pi - \theta) + sin^2\theta = 0;$$

Solving this equation, we get:

$$\theta_{optim} = 1.1rad = 63^0;$$

The radius vector of this point is:

$$r_{optim} = \frac{H}{\pi}\, x \frac{\pi - \theta_{optim}}{sin\theta_{optim}} = \frac{H}{\pi}\, x \frac{\pi - 1.1}{\sin - 1.1} = 0.7292H; \qquad 31$$

The height of the turbine installation at this point:

$$hust = r_{optim} \sin 1.1 = 0.65H; \qquad 32$$

The flow velocity at this point is:

$V_{AB^{opt}=1.27\, x\, ws,}$ Assuming 4m/s

That is, the flow rate increases by 27%.Diffuser augmented effect. Making an increment of 26.41%.

6 Conclusions

The increases in aerodynamic efficiencies were attainable because of the concept of diffuser augmented system (DAS). This improvement is primarily due to efficient extraction of energy (wind) blowing near the hub of the main rotor, but in part also due to the addition of another energy extracting device called diffuser augmented system. Calculation of the developed model showed the possibility of a significant increase in the power and efficiency of a modernized turbine (diffuser augmented wind turbine) in comparison with a classical turbine. (bare wind turbine) It is shown that the proposed modernization of the turbine increases the wind capture by 27%. 32% and 38%. To verify the performance of the DAWT, the simulation results have been demonstrated on the virtual platform of Matlab/Simulink.

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