APPLICATION OF QUEUEING THEORY TO PERFORMANCE CHARACTERIZATION OF A MEDIUM SCALE VEGETABLE OIL- PRODUCING PLANT IN OWERRI

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Abstract

The performance characterization of a medium scale vegetable oil plant in Owerri is presented. Using the queue methodology, it was established that the plant operates the queue model [(M_1/M_2/s);(infinite, F-C,F-S)] with Poisson arrival mass rate of palm kernel and Poisson processing rate of the machines and multiple but identical servers. Infinite population and strict queue discipline of First-Come, First-Served. The performance characteristics of the queue system of the plant; queue length, waiting time in queue, number of units in system and waiting time in the system were discovered to decrease with decreases in utilization factor or decrease in number of processing machines. As a result the total time spent in the system is dominated more by service time rather than waiting time in queue. At 0.745 utilization factor the waiting time in system is 1.86 days out of which 0.49 days is waiting time in queue. There is triple increase in waiting time in queue and queue length at any machine breakdown. This queue performance unveiled, equips the management with critical information that enables cost effective management of the queue problems of the plant.

Keywords: Queue, Performance Characteristics, Plant.

1.0 INTRODUCTION

In Nigeria, the vegetable oil industry emerged as a result of the 1970 – 1974 economic plans where industries whose raw materials can be sourced locally were encouraged by restricting the importation of such raw materials (Owuama et al., 2013). As a result, modern plants for vegetable oil production (small, medium and large scale) evolved with consequent sourcing of their raw materials locally and randomly.

The raw materials supplied in most cases, were not balanced with the processing rate causing queue formation at the storage facility. This is in accordance with Kolawole (1973), Loomba (1978) and Sharma (2007) who stated that queue results when calling units exceed the number of available servers.

Queueing theory is the study of waiting line (Sang et al., 1991). It is a mathematical evaluation providing optimal service to customers while reducing service cost as much as possible (Higgins, 1976) and (Sharma, 2007). The queueing models of the theory represent the various types of queueing systems in practice. The formula for each model indicates the performance characteristics on the variety of circumstances. In effect, queueing models form the major determinant on how to operate a queueing system effectively.

The performance characteristics of the system of queues include the waiting times in queue and in system, the queue length and number of units in the system (Sang et al., 1991) and (Sharma, 2007).

This research laid emphasis on the queue system of the store/production department of the plant. The plant's management has been operating in their various ways to minimize queue problems but to no avail. With this study, the decision parameters, otherwise called performance characteristics of the queue system in the plant, will be unveiled there-by leading to minimal cost due to queue in the plant.

1.1 *Objective*

The main objective of this research is performance characterization of the queue problems in the medium scale vegetable oil plant. The sub-objectives are; tests of data for Poisson distribution, identification of queue model, and calculation of the performance characteristics of the store/production department of the plant.

2.0 METHODOLOGY

The data requirement to achieve the research objectives includes the queue structure of the production system, the rate of palm kernel arrival per day, rate of palm kernel processing per day per machine and the number of processing machines engaged per day for the plant. The data obtained are analyzed statistically to obtain the mean values, $\bar{x} = \frac{(\sum \text{fx})}{N}; where\ \text{f} = \text{frequency}, \text{x} = \text{variable and } N = \text{sum of frequency}$ (Murray et al., 2006). Further analysis to test for fitness into a presumed Poisson distribution is carried out using chi square test. That is, $\chi^2 = (o_i - e_i)^2/e_i$; $v = k - 1 - m$;

where o_i= observed frequency, e_i = expected frequency, v = degree of freedom, k = number of terms in the formula for χ^2, m = number of quantity obtained from the observed data needed to calculate the expected frequency. (Murray et al., 2006).

If $X_\alpha^2 \geq \chi^2$ we cannot reject that the data is from a Poisson distribution; $\alpha = 0.05$ = significant level (Richard et al., 2011). The result of the test as well as the structure of the queue system enables model definition and assumptions. With the model equations for [($M_1/M_2/s$); (infinite F-C, F-S)], according to Sharma (2007) i.e.,

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s . \frac{s\mu}{s\mu - \lambda} \right]^{-1} \quad (1)$$

$$L_q = \left[\frac{1}{(s-1)!} . \left(\frac{\lambda}{\mu}\right)^s . \frac{\lambda\mu}{(s\mu - \lambda)^2}\right] P_0 \quad (2)$$

$$L_s = L_q + \frac{\lambda}{\mu} \quad (3)$$

$$W_q = \left[\frac{1}{(s-1)!} . \left(\frac{\lambda}{\mu}\right)^s . \frac{\mu}{(s\mu - \lambda)^2}\right] P_0 = \frac{L_q}{\lambda} \quad (4)$$

$$W_s = W_q + \frac{1}{\mu} = \frac{L_q}{\lambda} + \frac{1}{\mu} \quad (5)$$

$$\gamma_s = \frac{\lambda}{s\mu} \quad (6)$$

the performance characteristics of the queue system are predicted. In so doing, the decision parameters are unveiled, hence objectives achieved.

3.0 RESULT AND DISCUSSION

The results of the analysis of the information gathered and the discussion of the results are presented.

3.1 *Measured Data*

The model [($M_1/M_2/s$); (infinite F-C, F-S)] was applied to the medium scale vegetable oil producing plant in Owerrri. Sixty-day data collection was considered mainly in the production/store department of the plant. The calling unit (customer) in the plant is the palm kernel mass arriving the plant from various supply sources. The server is the machine processing the masses of palm kernel arriving the system. The queue structure indicates a queue system whereby the arriving masses of palm kernel come from infinite source and enter the storage facilities for processing in order of arrival, i.e., first-come first-served , (F-C, F-S). From the storage facility, they must be processed by the machines arranged in parallel, hence no reneging.

The mass of palm kernel both arriving and processed per day are classified/ unitized, and are presented in Tables 1 and 2.

Table 1: Frequency Distribution of Mass of Palm Kernel Arriving Per Day (Tonnes) in the Plant

Mass of Kernel Arriving Per Day (Tonnes)(M_λ)	Class Mark	Frequency (F)	Number of Variable, $X_\lambda = M_\lambda$ (New Equivalence, Unitized)
00.0 – 9.9	04.95	8	0
10.0 – 19.9	14.95	9	1
20.0 – 29.9	24.95	10	2
30.0 – 39.9	34.95	13	3
40.0 – 49.9	44.95	9	4
50.0 – 59.9	54.95	7	5
60.0 – 69.9	64.95	4	6

Table 2: Frequency Distribution of Mass of Palm Kernel Processed Per Day (Tonnes) in the Plant

Mass of Kernel Processed Per Day (Tonnes)(M_μ)	Class Mark	Frequency (F)	Number of Variable, $X_\mu = M_\mu$ (New Equivalence, Unitized)
00.0 – 9.9	04.95	6	0
10.0 – 19.9	14.95	14	1
20.0 – 29.9	24.95	15	2
30.0 – 39.9	34.95	14	3
40.0 – 49.9	44.95	9	4
50.0 – 59.9	54.95	2	5

Using the mean equation and Tables 1 and 2, the means of the mass of palm kernel processed per day per machine, and the mass of palm kernel arriving per day are calculated. Also, average number of machines processing per day is presented (See table 3).

Table 3: Mean Distributions of the Mass of Palm Kernel Arriving and Mass Processed When Unitized

Plant	Mean Mass Units of Palm Kernel Arriving Per Day (Λ_{plnt})	Mean Mass Units of Palm Kernel Processed Per Day (μ_{splant})	Mean Number of Machines Processing Per Day ($\dot{s}$)	Mean Rate of Processing Per Machine Unit Per Day (μ_{plant})	$\frac{\lambda_{plant}}{\mu_{plant}}$
C	2.72	2.20	3	0.73	3.699

NB: C = The plant.

Based on these values of the mean distributions the assumption of Poisson arrival rate and processing rate were tested by the Chi square test of goodness of fit, (see tables 4 and 5 below).

Table 4: Fitting a Poisson distribution to the Mass of Palm Kernel Arriving Per Day (Mean Distribution, Λ_c = 2.7).

Mass of Kernel Arriving Per Day (Tonnes)	Number of Variable, $X_\lambda = M_\lambda$, (New Equivalence, Unitized)	Pr{X Occurring}	Observed Frequency (O_i)	Expected Frequency (E_i)	$(O_i-E_i)^2/E_i$
(m_λ)					
00.0 – 9.9	0	0.067	8	4	4.0000
10.0 – 19.9	1	0.181	9	11	0.3636
20.0 – 29.9	2	0.245	10	15	1.6667
30.0 – 39.9	3	0.220	13	13	0.0000
40.0 – 49.9	4	0.149	9	9	0.0000
50.0 – 59.9	5	0.080	7	5	0.8000
60.0 – 69.9	6	0.036	4	2	2.0000

From table 3.4, χ^2 =.8.8303;. v = k-1-m = 7-1-1 = 5. Therefore, $x^2_{0.05} = 11.070$
Crt.: $x^2_{0.05} \geq \chi^2$, we cannot reject that the sample is from Poisson distribution.

Table 5: Fitting a Poisson distribution to the Mass of Palm Kernel Processed Per Day (Mean Distribution, μ_{sc} = 2.2)

Mass of Kernel Processed Per Day (Tonnes)	Number of Variable, $X_\mu = M_\mu$, (New Equivalence, Unitized)	Pr{X Occurring}	Observed Frequency (O_i)	Expected Frequency (E_i)	$(O_i-E_i)^2/E_i$
(m_μ)					
00.0 – 9.9	0	0.111	6	7	0.1429
10.0 – 19.9	1	0.244	14	15	0.0667
20.0 – 29.9	2	0.268	15	16	0.0625
30.0 – 39.9	3	0.197	14	12	0.33331
40.0 – 49.9	4	0.108	9	6	1.5000
50.0 – 59.9	5	0.048	2	3	0.3333

Also from table V, χ^2= 2.4387; v = k-1-m = 6-1-1 = 4. Therefore, $x^2_{0.05} = 9.488$
Crt. $x^2_{0.05} \geq \chi^2$, we cannot reject that the sample is from Poisson distribution.
From this test the arrival of the palm kernel into the plant fit very well into Poisson distribution. Processing rate of the palm kernel by the machines also fit into the distribution at 5% significant level.

3.2 *Calculated Data*

Sequel to the fitting into the Poisson distribution, the model equations (equations 1 – 6) are used to calculate the performance characteristics of the plant as shown in table 6.

Table 3.6: Calculated Performance Characteristics: Palm Kernel Processing Plant C .

(S)	(Γ)	(P_0)	(L_q)	(L_s)	(W_q)	(W_s)
0 – 3	$\frac{\lambda}{s\mu} > 1$					
4	0.932	0.0072	11.531	15.261	4.239	5.609
5	0.745	0.0192	1.326	5.056	0.488	1.858
6	0.621	0.0226	0.365	4.095	0.134	1.504
7	0.532	0.0236	0.114	3.844	0.042	1.412
8	0.466	0.0239	0.036	3.766	0.013	1.383
9	0.414	0.0240	0.011	3.741	0.004	1.374

where s = Number of processing machine units
γ = Utilization factor for processing units
P_0 = Probability of processing unit idle
L_q = Expected queue length
L_s = Expected number of palm kernel units in the system
W_q = Expected waiting time per palm kernel unit in queue
W_s = Expected waiting time per palm kernel unit in system

The characteristic relationship among the performance measures are depicted in the figures 1 – 5 below.

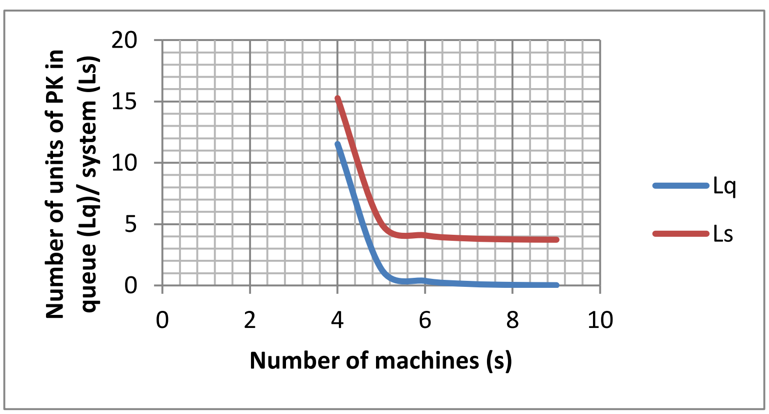

Figure 1: Relationship between s and Lq/Ls

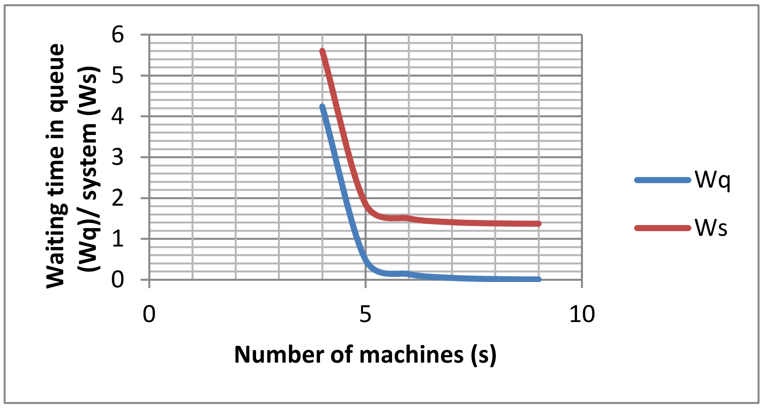

Figure 2: Relationship between s and Wq/Ws

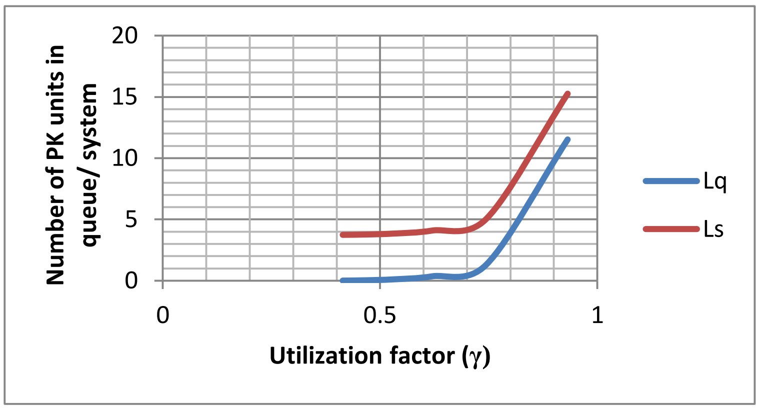

Figure3: Relationship between Γ and Lq/Ls

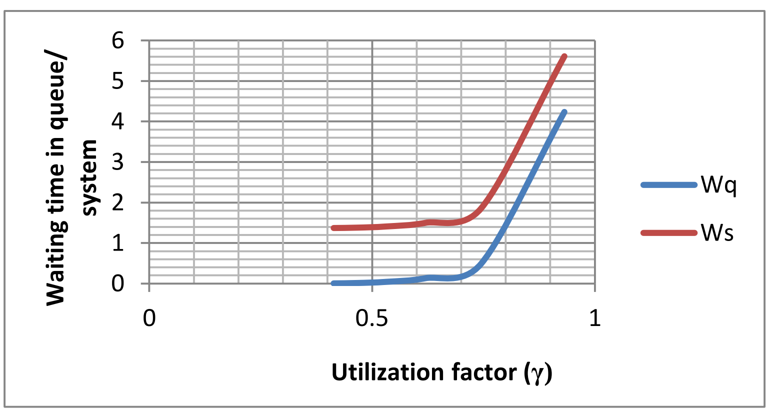

Figure 4: Relationship between Γ and Wq/Ws

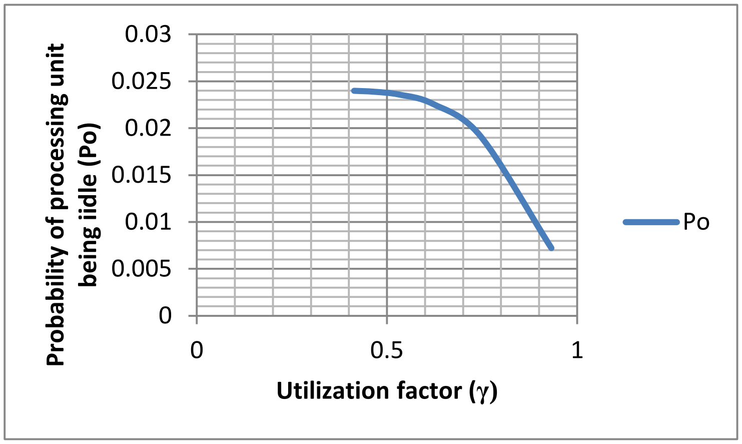

Figure 5: Relationship between Γ and Po

The results in Table 6 with λ_C = 2.72, μ_C = 0.73 and λ_C/μ_C = 3.73 are considered. It was observed that the increase in the number of processing units has decrement effect on the queue length, number of units of palm kernel in the system, waiting time in system and in queue. This is depicted in the figures 1 and 2. Also, the utilization factor which decreases with increase in number of machine has decrement effect on the queue length, and waiting time. see figures 3 and 4. Furthermore, utilization factor varies inversely with the probability of the processing machine being idle. This is depicted in figure 5.

The installed processing capacity of 7 units meets the stable state condition of $\lambda/s\mu < 1$, the operational average number of processing units of 3 exists. This is within the region of exploded queue hence the continuous building of queue observed i.e., 0 -3 processing units are unstable. Stability of the queue starts with 4 processing units. See figures 1 and 2, and table 6. The waiting time in the system is 1.411 days out of which 0.041 days is waiting time in queue and the queue length is 0.111 units of palm kernel. When 5 machines are processing, the waiting time in system is 1.86 days out of which 0.49 days are waiting time in queue. Beyond this number of machines, there is no significant change in waiting time in queue.

In general, as the utilization factor decreases the performance characteristics decrease. Also, breakdown of one processing unit increases the waiting time in the queue and queue length to about 3 times in the plant.

CONCLUSION

From the foregoing for the medium scale palm kernel oil producing plant in Owerri, it is concluded that;

- The queue model existing in the plant is [($M_1/M_2/s$); (infinite, F-C, F-S)], i.e., single queue, multiple but identical servers of mean service rate μ = 0.73 units per day per machine, infinite population and strict queue discipline first-come, first-served.
- As utilization factor decreases from 1, the queue length and waiting time in queue, units in the system as well as the waiting time in system decrease. The total time spent in the system is dominated more by service time rather than waiting time in queue.
- When utilization factor is 0.745, i.e., number of servers = 5, the waiting time in system = 1.86days out of which 0.49days are waiting time in queue.
- 0 – 3 units of machine operating is within unsteady state zone with exploded queue.

- Breakdown of a processing machine increases the queue length and waiting time in queue 3 times.

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